Determine all isosceles triangles with side lengths *a*, *b*, *c* such that

*a* = *y* *x*

*b* = *x* + *z*

*c* = *y* *z*

*x* + *y* + *z* < 10

Side lengths *a*, *b* and *c* must all be less than 8 and greater than 0. For any given side length *x*, where x , *x* ≥ 1 because *x* ≠ 0. Assuming side lengths *a*, *b* = 1, then *c*max = 7, as 1 + 1 + 7 = 9 < 10.

Integers *x*, *y* and *z* can be given similarly, with values less than 9. Side length *b* > *a*, *c*, as *b* is given by *x* + *z* as opposed to conjugate form *x* – *z*, similar to *a* = *y* – *x* or *c* = *y* - *z*. Assuming edge case *x* = 7, *z* = 1, b = 8.

The values of x, y and z can be computed such that *x*, *y* and *z* satisfy their given bounds. Given *a*, *b* < *c*, *a* + *b* > *c* in any proper triangle. Not all values a, b, c that satisfy these bounds are viable (fig. 1.1 – 1.3).

*a* + *b* < *c*

*a* + *b* > *c*

*a* + *b* > *c*

*b*

*b*

*a*

*a*

*b*

*a*

*c*

*c*

figure 1.1

figure 1.3

figure 1.2

*c*

A pruned exhaustive search approach can be taken when solving for viable *a*, *b*, *c* values. A general structure of this is given. The algorithm described below is of O(*n*3) time complexity.

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triangles = 0

remain = 9 (Remain is the total sum x + y + z can equal)

for i in remain (Iterates for every possible value 0 <= i < 9)

y = i + 1

remain – i

for j in remain (Iterates for the remaining values where x + y + z < 9)

x = j + 1

remain – j

for k in remain (Iterates for the remaining values where x + y + z < 9)

z = k + 1

a = y – x (Calculate a, b, c values)

b = x + z

c = y – z

if (a > 0) and (c > 0) (Checks if the side lengths are greater than 0)

sides = [a, b, c]

sort(sides)

if sides1 + sides2 > sides3 (Checks if a triangle exists)

if sides1 = sides2 or sides2 = sides3 (Checks if the triangle is isosceles)

triangles + 1 (Adds to the number of possible triangles)

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There are 12 possible triangles that satisfy the given specifications.

The viable triangles are the T, where every column indicates a sequence of possible a, b, c values.

### The corresponding x, y, z values are given by V, where every column indicates a possible sequence of x, y, z values.